Demography and Cross-country Savings Rates Differences: A New Approach and Evidence*

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January 8, 2008

Abstract

This paper quantitatively investigates the extent to which demography can explain the large differences in cross-country savings rates. The paper includes both dependency thesis, which is embodied in fertility rates, and pension motive for savings, which is captured by survival rates of the working-age agents. High fertility rates increase the expenditure burden of children and lower savings, while high longevity induces individuals to discount the future less heavily and consequently encourage savings. The two demographic factors are incorporated into an overlapping generations model and the steady state savings rates for a sample of 109 countries are computed. It shows that the two demographic factors can explain up to 68% of the dispersion in the cross-country savings rates. Furthermore, if the expenditure burden is sufficiently high, fertility has a greater impact on cross-country savings rates differences than longevity does. The model developed is also satisfactory in explaining the large gap in savings rates between the high and low income countries.

JEL classification: E21; J10; O16
Keywords: Saving, Fertility, Longevity

*I would like to thank Ravikumar, Raymond Riezman, Matthew Mitchell and the seminar participants at the University of Iowa and California State University Fullerton for useful comments. All remaining errors are mine.

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1 Introduction

It is a well known fact that savings rates are different considerably across country. In the period of 1980-2000, the average gross domestic savings rates in Benin and Burkina Faso were 1.0% and 2.6%, while Malaysia and Singapore were 36.0% and 45.1%, respectively. The mean and standard deviation of gross domestic savings rates for 109 countries during the period were relatively constant, around 19% and 9%, respectively.

The importance role of savings in economic development has been stressed in the literature. Neoclassical exogenous growth models suggest that an increase in savings rates generates higher growth in the short run, while endogenous growth models predict that higher savings can result in a permanent increase in growth rates (Barro, 1991; Levine and Renelt, 1992; Young, 1994). The strong correlation between savings and income levels is also a well-established empirical fact. For instance, Kelley and Schmidt (1996) find that savings rates are higher in countries with higher GDPs per capita, while Mankiw, Romer, and Weil (1992) find that savings rates (and population growth) explain up to 59% of the variation in international income.[1]

Of many factors considered that can affect cross-country savings rates differences, demographic change has attracted perhaps the most sizeable attention.[2] Yet, how demography change affects savings rates still remains an open question. Consider for example the effect of fertility rate. Ever since Coale and Hoover (1958), attempts to explain the link between fertility and savings have been extensively done. The thesis is, higher fertility increases the youth-dependency ratio which raises consumption and lowers saving. Empir-

[1]By replicating Mankiw, Romer, and Weil (1992) and using more recent data, I find that savings rates and labor growth rate explain about 48% of the variation in international income. Using Klenow and Rodriguez-Clare's (1997) variance decomposition, savings rates account for 23% of international income differences. Working along the lines of Prescott (1999), I find little evidence that savings rates explain international income differences. Given the results of these three approaches, I am inclined to conclude that the effect of savings rates on income levels is moderately important.

[2]Some countries have low savings rates because of institutions and policies that discourage savings and investments such as high effective tax rates on capital income (McKinnon, 1973; Easterly and Rebelo, 1993). The observed low/high savings might also be due to subsistence consumption needs (Gersovitz, 1988).

ical analyses, however, have been both supportive (Leff, 1969, 1984; Kelley and Schmidt, 1996) and critical (Bilsborrow, 1979; Goldberger, 1973; Ram, 1982) of the link. Similarly, increased longevity raises old-dependency ratio which then leads the higher consumption and health needs of the elderly which, in turn, lowers savings. Studies such as Leff (1969), Modigliani (1970), Graham (1987), Masson, et al. (1998, Higgins (1998), and more recently Li et al. (2007) find that old age-dependency ratio has negative relationship with savings, while Ram (1982, 1984) finds insignificant relationship.

This paper offers a different approach in explaining the effects of demography on cross-country savings differences. First, instead of modeling the effect of fertility on savings through the youth-dependency ratio, this study models dependency through child-rearing expenditures. High fertility rates are assumed to increase the expenditures on children and lower savings. Thus, the effect of fertility on savings depends upon the fraction of income spent on rearing an individual child. While it is similar in nature with the standard approach, this approach has an advantage. Rather than being constrained by the youth-dependency ratio, which is fixed by data, this approach allows for an investigation of the effect of fertility on savings for different plausible fractions of income spent on a child, which can offer a potential answer to the opposite results on the effect of fertility on savings as found by previous studies.

Secondly, unlike the standard approach that assumes the negative effect of longevity on savings, this paper considers longevity as an indicator of lifetime uncertainty that can positively affect savings. While previous studies have investigated the effect of lifetime uncertainty on savings (Yaari, 1965; Davies, 1981; Hurd, 1989), their main focus is to study the the effect of lifetime uncertainty on the dissaving of the elderly to reconcile the empirical findings which indicate that the elderly do not dissave as fast as the simple life-cycle model predicts (Mirer, 1979; Menchick and David, 1983). Instead, this paper focuses on the lifetime uncertainty of the working-age population.[3] Since savers in the population are generally in the labor force, changes in their survival rates will affect their consumption-savings profile. For instance, when workers' survival rates are low, they discount the future more heavily and are less inclined to save. Likewise, when workers' survival rates are high, they will have incentive to save for their future consumption.

The two channels, fertility rates and workers' survival rates, are incorporated into a four-period overlapping generations model in which agents begin their lives as children, progress through two periods of working age, and end in retirement. This setup is consistent with the evidence from household survey data where saving happens late in life, between the ages of 40 and 60, and young workers (aged 20-40 years) hardly save at all due to being constrained by youth-dependency effects. The steady state savings rates for a sample of 109 countries are computed and the analysis leads to the following results.

First, demographic factors explain up to 68% of the dispersion in the cross-country savings rates. Secondly, the fraction of household income spent on rearing a child plays an important role in determining which demographic factor that has a greater contribution in explaining the variation in the savings rates. When the fraction is sufficiently low, adult-survival rates have a greater contribution to the differences in cross-country savings rates; when it is sufficiently high, fertility rates have a greater contribution. These results perhaps can shed light on the different results of previous studies regarding the effect of youth dependency on savings. Thirdly, the result is relatively satisfactory in explaining the large differences in savings rates between the high and low income countries.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 presents the qualitative implications of the model, followed by the quantitative impli-

[3]Recent study by Bloom et al. (2003) adds longevity to a standard model of life-cycle saving and through their regression empirical tests, they found that the inclusion of life expectancy into savings regressions not only improves the fit, but also can explain the remarkable savings boom in East Asia, which is difficult to explain using standard life-cycle model. Unlike their work, I use survival rates, instead of life-expectancy, of the population aged 20-40 and 40-60 and calibrate the steady state savings rates for the countries in this study's datasets.

cations in Section 4. Conclusions are drawn in Section 5. Data, tables, and figures are in Appendix

2 The Model

Consider an economy populated by overlapping generations of four-period lived agents, a representative firm producing a single good, and a government. A continuum of agents are born each period and agents within a generation are identical, thus allowing for the consideration of a single representative agent for each period. The four periods of life are referred to as childhood, young age, middle age and old age. Throughout the paper, superscripts denote age cohorts, subscripts denote time periods, uppercase letters denote aggregate variables, and lowercase letters denote agent variables.

2.1 Demographics

Agents of generation s face random survival from age $t-1$ to t, denoted by variable π_t^s, $t = s+1, s+2, s+3$:

$$\pi_{s+j}^s = \begin{cases} 1 & \text{with probability} \quad p_j \\ 0 & \text{with probability} \quad 1-p_j \end{cases}$$

where p_j is exogenously given and $j = 1, 2, 3$. Agents born in period $t-1$ enter into the young age period and are capable of reproducing n_t children in period t. Let N_t^{t-1} be the total of agents at time t who were born at time $t-1$. The total number of agents born at time t is $N_t^t = n_t N_t^{t-1}$ and the total number of young agents at time $t+1$ is $\pi_{t+1}^t n_t N_{t+1}^t$. The middle-aged agents at time t will die with certainty in time $t+1$. The age distribution in period t consists of N_t^t child, N_t^{t-1} young-aged, N_t^{t-2} middle-aged, and N_t^{t-3} old-aged agents. Total population at time t is $N_t = N_t^t + N_t^{t-1} + N_t^{t-2} + N_t^{t-3}$ which equals

$$N_t = N_{t-2}^{t-3}\{n_t \pi_t^{t-1} n_{t-1} + \pi_t^{t-1} n_{t-1} \pi_{t-1}^{t-2} n_{t-2} + \pi_t^{t-2} \pi_{t-1}^{t-2} n_{t-2} + \pi_t^{t-2} \pi_t^{t-3}\}.$$

And total labor force is

$$L_t = N_t^{t-1} + N_t^{t-2} = N_{t-1}^{t-2}(\pi_t^{t-1} n_{t-1} + \pi_t^{t-2}).$$

The demographic structure depends both on fertility and survival rates. In the steady state with constant survival probability and fertility rates, the growth rates of the labor force and of population are both given by

$$\frac{L_{t+1}}{L_t} = \frac{N_{t+1}}{N_t} = 1 + g = p_1 n, \tag{1}$$

where p_1 is the probability of the children surviving to young age.

2.2 Production

Output is generated according to a constant returns to scale neoclassical production function $Y_t = AK_t^{\alpha} L_t^{1-\alpha}$, where A is the technological parameter, assumed to be constant over time, K_t is the aggregate capital stock and L_t is the number of workers.

Factors markets are efficient so that capital and labor are rewarded their marginal products:

$$w_t = (1-\alpha) A K_t^{\alpha} L_t^{-\alpha}, \tag{2}$$

$$r_t = \alpha A K_t^{\alpha-1} L_t^{1-\alpha} - \delta. \tag{3}$$

where w_t is the real wage, r_t is the real interest rate, and δ is the depreciation rate of capital. The aggregate stock of capital evolves according to $K_{t+1} = I_t + (1-\delta)K_t$, where I_t is aggregate investment. Since aggregate savings are the aggregate output minus aggregate consumption, $S_t = I_t$.[4]

[4] I assume a closed economy. Theoretically, in an open economy where capital is internationally mobile, domestic saving rates would not necessarily be correlated with domestic investment rates. However,

2.3 Preferences

In childhood, agents are not active decision makers and they rely on their parents, young workers, for consumption. Each of the young and middle-aged agents, referred here as working-age agents, is endowed with one unit of time which is inelastically supplied to the labor force and receives wage w_t. The young workers decide how much to consume (c_t), save (s_t) and spend on child consumption. The child-rearing expenditure takes the form $\gamma n w_t$, where $0 < \gamma < 1$ is the fraction of income for rearing a child and n is an exogenous number of children per young worker. Reproduction only takes place during the young age. The budget constraint facing young workers at time t is

$$c_t^{t-1} = (1 - \gamma n) w_t - s_t^{t-1}. \tag{4}$$

This feature of the model permits a more accurate depiction of saving behavior over the life cycle. The link between fertility and savings allows for the possibility that lower youth dependency from declining fertility may positively affect aggregate saving. An exogenous reduction in fertility would motivate parents to substitute more of their resources into savings and wealth accumulation in other forms for their future consumption. Other things being equal, a lower fertility implies higher savings rates.

The government enters the economy for the purpose of collecting savings from the deceased working-age agents of age cohort $t-1$ at time t and equally distributing them in a lump-sum transfer to the surviving agents of the same cohort at period $t+1$.[5] Thus,

the correlation between national savings and investment rates across OECD countries, where capital is relatively free to move, is one of the best-established facts in international economics. Empirical evidence suggests that the high correlation is robust to changes in time period. This indicates that, on average and over long periods of time, changes in capital accumulation respond mostly to changes in domestic savings (See for example Feldstein and Bacchetta, 1992). For less developed countries which are less open, the correlation between investment and savings rates is expected to be higher. Therefore, the closed-economy assumption is not unreasonable.

[5] Alternatively, I can assume that there is a perfect annuities market whereby all savings are intermediated through mutual funds which invests the savings and guarantees a gross return R_{t+j} to the surviving agents of $t-1$ at period $t+j$, ($j = 1, 2$) as in Blanchard (1985) and Yaari (1965). In both cases, the gross return on capital is given by R_{t+j}/p_j and the results will be similar to the scheme introduced in

the budget constraint of a period $t+1$ middle-aged agent is

$$c_{t+1}^{t-1} = w_{t+1} + R_{t+1}s_t - s_{t+1}^{t-1} + \triangle_{t+1}, \tag{5}$$

where w_{t+1} is wage income, s_t and s_{t+1} are the savings, $R_{t+1} = 1 + r_{t+1} - \delta$ is the gross return on capital held from period t into $t+1$, and $\triangle_{t+1}$ is the transfer. The total savings of those who die before reaching middle age is $N_t^{t-1}(1-p_2)R_{t+1}s_t^{t-1}$ while the total transfer for the surviving young agents is $N_{t+1}^{t-1}\triangle_{t+1}$. Hence, the transfer for each surviving young worker is $p_2\triangle_{t+1} = (1-p_2)R_{t+1}s_t^{t-1}$.

The budget constraint for a period $t+2$ retiree is simply:

$$c_{t+2}^{t-1} = R_{t+2}s_{t+1}^{t-1} + \triangle_{t+2}, \tag{6}$$

where R_{t+2} is the return on capital held from period $t+1$ into $t+2$, and the transfer for the old agents is $p_3\triangle_{t+2} = (1-p_3)R_{t+2}s_{t+1}^{t-1}$.

Preferences are additively separable across time of the CRRA class utility function with risk aversion σ. The expected discounted lifetime utility of an agent born in period $t-1$ is given by

$$U(c_t^{t-1}, c_{t+1}^{t-1}, c_{t+2}^{t-1}) = \frac{(c_t^{t-1})^{1-\sigma}}{1-\sigma} + \beta p_2\Big[\frac{(c_{t+1}^{t-1})^{1-\sigma}}{1-\sigma} + \beta p_3\frac{(c_{t+2}^{t-1})^{1-\sigma}}{1-\sigma}\Big], \tag{7}$$

where c_t^{t-1}, c_{t+1}^{t-1}, and c_{t+2}^{t-1} are the consumption levels of a generation $t-1$ agents in period t, $t+1$ and $t+2$. β is the discount factor, where $0 < \beta < 1$. In the context of a cross-country study, β can be viewed as the common world discount factor, while survival rates p_2 and p_3 are stochastic discount factors specific to individual country.

Each worker of generation $t-1$ maximizes the expected lifetime utility in (7) subject to the budget constraints $(4) - (6)$. The optimal young worker behavior is described by

this paper.

the Euler equations:

$$(c_t^{t-1})^{-\sigma} = \beta p_2 R_{t+1} (c_{t+1}^{t-1})^{-\sigma}, \tag{8}$$

$$(c_t^{t-1})^{-\sigma} = \beta^2 p_2 p_3 R_{t+1} R_{t+2} (c_{t+2}^{t-1})^{-\sigma}. \tag{9}$$

2.4 Equilibrium

Definition 1 *Given exogenous survival rates p_j, exogenous fertility rates n, and the initial capital stock K_0, a competitive equilibrium is a set of household allocations $\{c_t^{t-j}, s_t^{t-j}\}_{t=0}^{t=\infty}$, aggregate input $\{K_t, L_t\}_{t=0}^{t=\infty}$, real prices for the factors of production $\{r_t, w_t\}_{t=0}^{t=\infty}$ such that:*

i. Young workers maximize utility in (7) subject to $(4)-(6)$.

ii. Firms maximize profits for each period in which returns to factors are as described in equations (2) and (3).

iii. The labor market clears: $L_t = N_t^{t-1} + N_t^{t-2}$.

iv. The market for goods clears:

$$\sum_{j=1}^{3} N_t^{t-j} c_t^{t-j} + K_{t+1} = F(AK_t, L_t) + (1-\delta)K_t. \tag{10}$$

v. The market for capital clears:

$$K_{t+1} = N_t^{t-1} s_t^{t-1} + N_t^{t-2} s_t^{t-2}. \tag{11}$$

3 Demography and Capital Accumulation

Aggregate savings is the sum of the savings of the young, middle-aged and old agents. Hence, aggregate savings at time t, S_t equals $N_t^{t-1}s_t^{t-1} + N_t^{t-2}s_t^{t-2} + N_t^{t-3}s_t^{t-3} = K_{t+1} -$

$(1-\delta)K_t$. And the savings rate is given by

$$\frac{S_t}{Y_t} = \frac{1}{A}\Big[(1+g)\frac{k_{t+1}}{k_t} - 1 + \delta\Big]k_t^{1-\alpha}. \tag{12}$$

Assuming technological parameter A, capital share α, and depreciation rate of capital, δ, are the same across countries, the cross-country savings rates are determined by the labor force growth rate, g, the evolution of capital, $\frac{k_{t+1}}{k_t}$, and the level of capital stock, k_t. The last of these is affected by fertility and survival rates.

Let us examine how fertility and survival rates affect capital accumulation. The optimal consumption profile over the life cycle is given by

$$c_t^{t-1} = (1-\gamma n - \frac{\Phi_1}{\Phi_2})w_t + \frac{1}{\Phi_2}w_{t+1}, \tag{13}$$

$$c_{t+1}^{t-1} = \Big(\beta p_2 R_{t+1}\Big)^{\frac{1}{\sigma}} c_t^{t-1}, \tag{14}$$

$$c_{t+2}^{t-1} = \Big(\beta^2 p_2 p_3 R_{t+1} R_{t+2}\Big)^{\frac{1}{\sigma}} c_t^{t-1}, \tag{15}$$

where

$$\Phi_1 = (1-\gamma n)\Bigg[\frac{p_3\Big(\beta^2 p_2 p_3 R_{t+1} R_{t+2}\Big)^{\frac{1}{\sigma}}}{R_{t+2}} + \Big(\beta p_2 R_{t+1}\Big)^{\frac{1}{\sigma}}\Bigg],$$

$$\Phi_2 = \frac{R_{t+1}}{p_2} + \Big(\beta p_2 R_{t+1}\Big)^{\frac{1}{\sigma}} + \frac{p_3\Big(\beta^2 p_2 p_3 R_{t+1} R_{t+2}\Big)^{\frac{1}{\sigma}}}{R_{t+2}}.$$

Without loss of generality, assume a log utility function ($\sigma = 1$). From the optimal consumption profile in (13)$-$(15), the savings of the young and middle-aged agents at time t are given respectively by

$$s_t^{t-1} = \lambda_1(n, p_2, p_3)w_t - \lambda_2(n, p_2, p_3)\frac{w_{t+1}}{R_{t+1}},$$

$$s_t^{t-2} = \lambda_3(n, p_2, p_3) R_t w_{t-1} - \lambda_4(n, p_2, p_3) w_t,$$

where

$\lambda_1 \lambda_5(n, p_2, p_3) = (1 - \gamma n)\left[\beta p_2^2 + (\beta p_2 p_3)^2\right],$

$\lambda_2 \lambda_5(n, p_2, p_3) = p_2,$

$\lambda_3 \lambda_5(n, p_2, p_3) = (1 - \gamma n)\beta^2 p_2 p_3^2,$

$\lambda_4 \lambda_5(n, p_2, p_3) = (\beta p_2 p_3)^2,$

$\lambda_5 = \left[1 + \beta p_2^2 + (\beta p_2 p_3)^2\right].$

It is straigthforward to show that

$$\frac{ds_t^{t-1}}{dn} < 0, \frac{ds_t^{t-1}}{dp_2} > 0, \frac{ds_t^{t-1}}{dp_3} > 0, \frac{ds_t^{t-1}}{dw_{t+1}} < 0, \text{ and } \frac{ds_t^{t-1}}{dR_{t+1}} > 0.$$

And,

$$\frac{ds_t^{t-2}}{dn} < 0, \frac{ds_t^{t-2}}{dp_2} > 0, \frac{ds_t^{t-2}}{dp_3} > 0.$$

That is, the savings rates of the young and middle-aged agents are decreasing in fertility and increasing in survival rates. This is, however, just the partial effect of a change in fertility and survival rates for given wages and interest rates. An increase in savings will increase the next period capital stock (k_{t+1}), raise wages and then reduce the interest rate, which in turn, affect savings.

Expressing the capital market condition in (11) in terms of capital per worker yields

$$(1 + g)(p_1 n + p_2) k_{t+1} = p_1 n s_t^{t-1} + p_2 s_t^{t-2}. \tag{16}$$

The asset market equilibrium condition in (16) can be used to derive the effects of fertility and survival rates on the accumulation of capital stock at time $t + 1$, k_{t+1}, for fixed k_t. Totally differentiating (16) and setting $dp_2 = dp_3 = 0$ yield

$$\frac{dk_{t+1}}{dn} = \frac{p_1[s_t^{t-1} - (1+g)k_{t+1}]}{(1+g)(p_1n+p_2) - p_1n\left[\frac{ds_t^{t-1}}{dn} - \frac{ds_t^{t-1}}{dw_{t+1}}k_{t+1}f''(k_{t+1}) + \frac{ds_t^{t-1}}{dR_{t+1}}f''(k_{t+1})\right]}.$$

The denominator is positive since $f''(k_{t+1}) < 0$. The numerator can be written as $\frac{p_1p_2}{p_1n+p_2}(s_t^{t-1} - s_t^{t-2})$. Since $s_t^{t-1} - s_t^{t-2} < 0$, $\frac{dk_{t+1}}{dn} < 0$. That is, a reduction in the fertility rate increases capital accumulation, raises wages and lowers interest rates.

The effects on capital stock accumulation of the survival rate the young to middle-aged and from the middle-aged to old periods are is described by the followings:

$$\frac{dk_{t+1}}{dp_2} = \frac{\frac{p_1n}{p_1n+p_2}\left(s_t^{t-2} - s_t^{t-1}\right) + p_2\frac{ds_t^{t-2}}{dp_2}}{(1+g)(p_1n+p_2) - p_1n\left[\frac{ds_t^{t-1}}{dp_2} - \frac{ds_t^{t-1}}{dw_{t+1}}k_{t+1}f''(k_{t+1}) + \frac{ds_t^{t-1}}{dR_{t+1}}f''(k_{t+1})\right]} > 0,$$

$$\frac{dk_{t+1}}{dp_3} = \frac{1}{(1+g)(p_1n+p_2) - p_1n\left[\frac{ds_t^{t-1}}{dp_3} - \frac{ds_t^{t-1}}{dw_{t+1}}k_{t+1}f''(k_{t+1}) + \frac{ds_t^{t-1}}{dR_{t+1}}f''(k_{t+1})\right]} > 0,$$

since $s_t^{t-2} > s_t^{t-1}$. In both cases, the next period capital stock is increasing in survival rates, p_2 and p_3.

3.1 Steady-state Savings Rates Relations

A steady state for the dynamics of equation (16) is a level $\hat{k}$ such that

$$(1+g)(p_1n+p_2)\hat{k} = p_1ns_1(f(\hat{k}) - \hat{k}f'(\hat{k}), f'(\hat{k}), n, p_2, p_3) + p_2s_2(f(\hat{k}) - \hat{k}f'(\hat{k}), f'(\hat{k}), s_1(f(\hat{k}) - \hat{k}f'(\hat{k}), f'(\hat{k}), n, p_2, p_3), n, p_2, p_3),$$

where $\hat{k}$ is the steady state level of capital stock. The steady state savings rate is

$$\frac{S}{Y} = \hat{s} = \frac{1}{A}(g+\delta)\hat{k}^{1-\alpha}. \tag{17}$$

The following qualitatively summarizes the relationship between savings rates and demographic factors.

Results 1 *Steady state savings rate is decreasing in fertility rate.*

Proof.

$$\frac{d\hat{s}}{dn} = \frac{1}{A}(1-\alpha)(g+\delta)\hat{k}^{-\alpha}\frac{d\hat{k}}{dn} < 0.$$

■

Results 2 *Labor force growth rate has ambiguous effect on the steady state savings rate.*

Proof. From the capital market condition in equation (16), it is straightforward to show that $d_{k+1}/dg < 0$. Thus,

$$\frac{d\hat{s}}{dg} = \frac{1}{A}\hat{k}^{-\alpha}\left[\hat{k} + (1-\alpha)(g+\delta)\frac{d\hat{k}}{dg}\right] \leq \text{or} \geq 0.$$

■

Higher labor growth rate may decrease capital stock, but it can also increase savings rates due to its effect on the age structure of the population. A lower labor growth rate increases youth dependency and lowers capital stock. This is captured by $p_1 n$ in equation (16). At the same time, higher labor growth rate increases savings rate by raising the proportion of workers relative to dissaving old agents. This is captured by g in equation (17). If the former outweighs the latter, an increase in labor force growth rate decreases the savings rate. In an attempt to investigate the effect of demography on aggregate savings, Krueger (2004) shows that a lower population growth rate may increase per capita saving in the short run, but reduces it in the long run. As the model here implies, in the long-run, savings rates may decrease or increase.

Results 3 *Steady state savings rate is increasing in the survival probability of the young to reach the middle-aged period (p_2) and of the middle-aged agents to reach the old period*

(p_3).

Proof.

$$\frac{d\hat{s}}{dp_2} = \frac{1}{A}(1-\alpha)(g+\delta)\hat{k}^{-\alpha}\frac{d\hat{k}}{dp_2} > 0.$$

and

$$\frac{d\hat{s}}{dp_3} = \frac{1}{A}(1-\alpha)(g+\delta)\hat{k}^{-\alpha}\frac{d\hat{k}}{dp_3} > 0.$$

■

4 Calibration

The central question I would like to investigate is whether a steady state of equilibrium in this model can produce savings rates consistent with the observed large difference in cross-country data. To obtain the steady state savings rate for each country, I first computed the steady state capital stock per worker, which is determined through the following non-linear equations:

$$(1+g)(p_1 n + p_2)k = p_1 n s^y + p_2 s^m, \tag{18}$$

$$s^y = \frac{p_2\Big\{(1-\gamma n)(\beta p_2 \hat{R})^{1/\sigma}\big[\hat{R} + p_3(\beta p_2 \hat{R})^{1/\sigma}\big] - \hat{R}\Big\}}{\hat{R}^2 + (\beta p_2 \hat{R})^{1/\sigma}\big[\hat{R} + p_3(\beta p_2 \hat{R})^{1/\sigma}\big]}(1-\alpha)A\hat{k}^{\alpha}, \tag{19}$$

$$s^m = \Big[1 - (1-\gamma n)(\beta p_2 \hat{R})^{1/\sigma}\Big](1-\alpha)A\hat{k}^{\alpha} + \Big[\frac{\hat{R}}{p_2} + (\beta p_2 \hat{R})^{1/\sigma}\Big]s^y, \tag{20}$$

where s^y is the savings of the young worker, s^m is the savings of the middle-age worker, $\hat{R} = 1 + \alpha A\hat{k}^{\alpha-1} - \delta$, and given $\hat{k}$, the savings rate for each country is given by equation (17).

For computation, it is assumed that each period of the model corresponds to 20 years. The relative risk-aversion parameter, σ, is set at 1.1, which indicates that higher interest rates lead to a moderate increase in savings rates. Empirical studies suggest a weak interest rate elasticity of aggregate domestic savings. For instance, Boskin (1978) finds a very low elasticity for the United States and Giovannini (1983) failed to find a positive effect of interest rate changes on private savings for developing countries. As McKinnon (1991) acknowledged, aggregate savings, as measured in the GNP accounts, do not respond strongly to higher real interest rates. For capital share in the production of output (α), a value of 0.33 is used. This number is close to Gollin's (2002) careful calculation of labor shares in which the shares for most countries fall in the range of 0.65 to 0.80. The depreciation rate of physical capital (δ) is set at 5% per year so that 65% of the capital stock is lost each period. Fertility rates and the labor-force-growth rates are taken from the World Development Indicators (World Bank, 2004). The child and adult survival rates (p_1, p_2 and p_3) are constructed from the life table published by the WHO (detailed explanations of the data are provided in the appendix).

The fraction of income allocated for rearing an individual child (γ) is set at 10%. This number, which is the average fraction across different income levels and household sizes, seems to be plausible. Based on the 1984, 1988-89, and 1993-94 Household Expenditure Surveys conducted by the Australian Bureau of Statistics, the child-rearing costs for low, middle, and high income groups are about 20%, 30%, and 40% of the total household expenditure, respectively. In the United Kingdom, the cost of raising one, two, and three children are about 15%, 20%, and 25%, respectively (Dickens, Fry, and Pashardes, 1996). Based on household surveys in the United States, the fraction of household income spent on children is roughly 20% for husband-wife families (Lino, 2004). The data from Haveman and Wolfe (1995) also shows that about 14% of gross domestic income is allocated for children expenditures in the US. The complete parameter values are given in Table 1.

4.1 Quantitative Implications

Given the parameter values in Table 1, the model is calibrated in such a way that the cross-country average of the savings rates in the model matches the data. My goal is to obtain variance of model predictions as close as possible to that of the data, keeping the mean of savings rates in both model and data equal.[6] The parameter used to control the average is the common discount factor β, which is set equal to 0.82 (it corresponds to 0.9903 per year) to match the mean savings rate of 18.7% in the data. Table 2 provides the summarized statistics of the data and the model. The standard deviation of the model predictions is about half of that observed in the data, indicating that the model predictions are less varied than the data.

I am interested in evaluating how much of the dispersion in the data is explained by the model. To assess that, I use the coefficient of determination, R^2. The value of R^2 in the model (0.2) suggests that it explains about 20% of the variation in the data. An alternative measure is inter-percentile ratio, which is less sensitive to outliers. Let us define d_p as the value of the p^{th} percentile of the distribution of d. Inter-percentile ratio compares what the 90^{th} to 10^{th} percentile ratio would be in the model to the actual value. In the data, the value of d_{90}/d_{10} is 5.4 while in the model it is 1.9. Using this measure, the model explains about 35% of the dispersion in the data.

The comparison between the distributions of the data and model predictions is shown in Figure 1. As the figure shows, the model can capture the lower tail of the distribution and less atisfactory in capturing the upper tail. While 95% of the predictions fall within the range of 10% to 30%, about 70% of the data occurred in this range. Figure 2 describes where the model fails in predicting cross-country savings rates. The model tends to under predict savings rates for certain countries, such as Botswana, Gabon, China and

[6]Controlling for the mean of the model predictions is important because although one can generate predictions on savings rates such that its standard deviation is close to that of data, its mean can be remotely distant from the data mean which is not the objective of this study.

Singapore. In these countries, central government savings constitute at a large percentage of gross domestic savings rates. Figure 2 also shows that the model tends to overpredict savings rates for countries, such as Israel, El Salvador, Benin and Madagascar, where governments are dissaving. As an alternative to gross domestic savings rates, I use private savings rates which are gross national savings minus central government savings (further detail is provided in the appendix) whose mean and variance are lower than those of the gross domestic savings rates.

To draw model implication for private savings rates, the parameter values are set the same as in Table 1, except that β is set equal to 0.74 (which corresponds to a discount factor of 0.985 per year). Table 3 summarizes the results. The exclusion of government savings improves the prediction of the model. The R^2 increases from 0.2 to 0.25 and the inter-percentile ratio also increases from 35% to 50%.

Compared to gross domestic savings rates, the model is relatively better at capturing the distribution of private savings rates (Figure 3). However, it is still less satisfactory in capturing the lower and upper tails of the distribution as model predictions are concentrated within the interval of 10% to 25%.

4.2 Alternative Parameter Values

When compared to both data sets (gross domestic and private savings rates), the model tends to overpredict savings rates for countries with high fertility. Suppose we categorize "overprediction" as the case where the model prediction is at least 5% higher than the actual savings rates. Thirty-two countries fall into this category and 22 of them have a relatively high fertility rate with an average rate of 2.5. To take this pattern into account, I explore the model prediction under a different parameter value of γ (the fraction of labor income allocated for rearing an individual child). This is intended to capture the lower savings rates for countries with high fertility and thereby increase the standard deviation of the model predictions. I set γ equal to 0.15, which seems to be plausible. Studies have shown that the fraction of income allocated for rearing a child in developed countries is about 10%. For developing countries, this fraction is likely to be higher. Since developing countries constitute more than a half of the countries in this study, it is reasonable to assign $\gamma > 0.1$. The remaining parameter values are the same as listed in Table 1, except the discount factor β is set at 0.94 (corresponds to 0.997 per year) to match the average of the model to that of data. The results in Table 4, labeled under Alternative 1, indicate that R^2 and the inter-percentile ratio increase from 0.2 to 0.25 and from 0.35 to 0.5, respectively, impliying that the model explains about 24% to 50% of the variation in the data.

While the model under the alternative parameter value of γ can generate a higher savings rate compared to the baseline parameter values, it still fails to account for the high savings rates in countries with high survival rates. In Result 3, it is shown analytically that the effects of survival rates (p_2 and p_3) on savings rates are higher for a larger depreciation rate of capital. Suppose the depreciation rate of physical capital is doubled from 0.05 to 0.1 per year (which is equivalent to 0.878 for a 20-year period). The results in Table 4, labeled under Alternative 2, show that neither R^2 nor inter-percentile ratio change relative to Alternative 1.

However, under the alternative parameter values, the model predictions on private savings rates are relatively better (Table 5). For instance, R^2 increases from 0.25 to 0.40 and the inter-percentile ratio also increases from 1.9 to 2.7, suggesting that the model can explain about 40% to 68% of the variation in the data. The results on R^2 are relatively comparable to previous empirical studies that link demographic factors and savings rates. The studies typically regress demographic factors, along with other variables such as income per capita and economic growth, on gross domestic savings rates. Leff (1969) and Ram (1982) show R^2 to be in the range of 0.4 and 0.6, while a study by Kelley and

Schmidt (1996) results in a R^2 range of between 0.3 and 0.63. Using data on private savings, Masson, Bayoumi and Samiei (1998) find that the value of R^2 is around 0.40.

4.3 Source of Variation: Fertility or Survival Rates?

Where does the source of variation in the model? Is it due to the variation in the survival rates or fertility rates? Here, I compare two special cases. The first is the case where the only factor affecting savings rates in the model is fertility (fertility only model). The second case is when the survival rates of the young and middle-aged agents are the only factors affecting savings rates in the model (survival only model). In the fertility only model, I set $p_2 = p_3 = 1$, while in the survival only model, γ is set equal to 0, and a common fertility is set at $n = 1.5$. The remaining parameter values are the same as in the Table 1 except for β which is set at 0.72 for the survival only model and 0.74 for the fertility only model, respectively.

Under baseline parameter values, the results in Table 6 suggest that survival rates explain most of the variation in the model. In addition, as Figure 4 shows, the distribution of savings rates in the survival only model is relatively better at capturing the distribution of the data. However, when the fraction of household income spent on rearing an individual child is increased from 10% to 15%, fertility rates are more important than adult survival rates in explaining savings rates differences. These results perhaps can shed a light on the pro and contra results regarding the effect of youth dependency on savings. In his regression analysis, Left (1969) finds that youth dependency is significant, while Ram (1982), by replicating Left (1969) and using new datasets, finds that the youth-dependency ratio is insignificant.

4.4 Savings Rates by Income Levels

The positive correlation between income and survival rates and the negative correlation between income and fertility rates as observed in the data suggest that the model should predict high savings rates for high-income countries and low rates for low-income countries. I use the model to investigate whether the two demographic factors can explain the large difference in the savings rates between high and low income countries. In the data, the average savings rate of the 15 richest countries is 23.5% while that of the 15 poorest countries is 8.9%. First, using the same baseline parameter values as listed in Table 1 and setting $\beta = 0.94$, I compute savings rates for the 15 richest countries to match its average savings rate of 23.5%. Next, using these parameter values, I compute savings rates for the 15 poorest countries and the resulting average is 13.6%. The difference between the predicted and the actual averages is still relatively large. One potential reason is that, on average, fertility rate in the 15 poorest countries (3.13) is more than three times the 15 richest countries (0.85). To weight this large difference in fertility, I investigate the model prediction when the fraction of income spent on a child is increased from 10% to 15%. With similar steps as in baseline parameter case and by setting $\beta = 0.785$, the average prediction of the savings rates for the 15 poorest countries is 10.4%, which is relatively close to the data. This shows that the model can satisfactorily explain the large difference in savings rates between the richest and poorest countries.

While the model can predict the large gap in savings rates of the richest and poorest countries relatively well, it is less satisfactory in generating sufficiently low or high savings rates to match the rates observed in the data. Recall the parameter listed in Table 1. The parameter values of fertility rate, labor growth rate, and survival probabilities are specific to each country and fixed by data. Other parameters such as σ, γ, and β can be chosen so that the predicted savings rates match the data. For example, by setting σ and β are set equal to 0.2 and 1.0, respectively, and given the values of other parameters listed in Table 1, the model can generate a savings rate of 38% for China as observed in the data. The model can also produce the observed savings rate of 3.7% for Madagascar when σ and β are set 6.0 and 0.65, respectively. It is reasonable to expect different σ and β across countries, as Lawrance (1991) argues that some countries have low savings rates because of higher subjective rates of time preferences, which implies less willingness to save. Since the objective of this study is to investigate the effect of demographic factors on savings rates, I abstract from the potential difference in the subjective rates of time preferences by assuming all countries have the same rates.

4.5 Case Study

This exercise shows that the model is less satisfactory in producing sufficiently low and high savings rates to match the observed rates for some countries without altering the cross-country average of savings rates. To describe this assertion, I examine the savings rates in two countries, Malaysia and South Africa, for which the trends of savings rates and demographic change were different during the 1970-2000 period. From 1970 to the mid 1980s, gross domestic savings rates in both countries displayed almost similar trends and levels. However, since 1986 the trends have diverged: savings rates increased in Malaysia and decreased in South Africa. Moreover, as shown in Figure 4, by the year of 2000 the difference in the savings rates between the two countries was about 30%! With respect to demographic factors, fertility rates in both countries have decreased by almost the same rate, while the trends for life expectancy, a proxy for survival rates, were about the same until 1990, and then decreased in South Africa and increased in Malaysia. Figure 5 shows that by year of 2002, the difference in life expectancy between the two countries was 26 years!

Can the model generate savings rates to closely match with the data for both countries? Let us suppose $\sigma = 0.5$, $\beta = 0.955$, and the remaining parameter values are the same as listed in Table 1. Given these parameter values, the model can generate the observed savings rates of 36% for Malaysia and 25.7% for South Africa, which is slightly higher than the data. If the value of σ is assumed to be the same for both countries and β is set equal to 0.885 to match the saving rates in South Africa (23.6%), the predicted savings rate for Malaysia is 33.9%. When $\beta = 0.92$ (equivalent to a discount factor of 0.996 per year), the predicted savings rates for both countries are 35% and 25%, which are very close to the data. However, with $\beta = 0.92$, the average of savings rates in the model for all countries in the dataset will be much higher than that of the data. This suggests that, while demographic factors can to some extent explain savings rates differences, other factors might also contribute to the differences. In the case of Malaysia, one such factor is macroeconomic policy and in South Africa, the factor is continuous shock in consumption due to high high seroprevalence of HIV/AIDS.

The high savings rates in Malaysia can also be linked to the country's financial liberalization and deepening in the 1980s.[7] In the case of South Africa, the high seroprevalence or the percentage of persons infected with HIV/AIDS might have contributed to the decline in the savings rates. According to a report by UNAIDS (2000), the seroprevalence was estimated at 19.94% at the end of 1999, which was the fifth-highest level of seroprevalence on the continent. The impact of such a pandemic is enormous economically and demographically. AIDS deaths lead directly to a reduction in the number of workers available and a permanent loss of income. Since these deaths occur to workers in their most productive years, it will affect overall economic output, including capital accumulation. In addition, the AIDS pandemic may have substantially increased household expenditures for medical expenses. This may explain why private health expenditure in South Africa is more than three times higher than in Malaysia. One study estimated the total direct and indirect costs due to HIV/AIDS could increase from 0.8% in 1991 to

[7] See for example Bank Negara Malaysia (2003), "Improving the Allocation of Domestic Savings for Economic Development: Case Study for Malaysia."

75% of total health expenditure in 2005 which translates to about 8% of current GNP by 2005 (Broomber, *et al*, 1993). The combination of greater health care expenditures and a loss of worker income due to AIDS can cause a significant drop in savings and capital accumulation. A sharp decrease in gross domestic saving in South Africa was due to the significant decrease in the gross private savings during the period. During the period of 1980-1999, gross private savings decreased from 28% to 15% (Prinsloo, 2000) which would continue to decrease if South Africa does not develop a national policy against HIV/AIDS (Freire (2004).

5 Conclusions

In this paper, I revisit the old debate on the effects of demography on cross-country savings rates and offer a new approach in studying the subject. One important feature of this paper is that the model includes both dependency thesis, which is embodied in fertility rates, and pension motive for savings, which is captured by survival rates of the working-age agents. The two demographic factors are incorporated into an overlapping generations model and the steady state savings rates for a sample of 109 countries are computed. The main results of the paper are as follows. First, the two demographic factors can explain up to 68% of the dispersion in the cross-country savings rates. The model performs better in explaining the variation in the private savings rates than gross domestic savings rates. Second, if the fraction of income spent on rearing a child is sufficiently high, fertility rates are shown to be more important than survival rates in explaining differences in cross-country savings rates. Conversely, if the fraction is sufficiently low, survival rates are shown to be more important. Third, while the model is satisfactory in explaining the large gap in savings rates between the high and low income countries, it is less satisfactory at capturing the tails of the cross-country savings rates distribution. One possible reason is that the high expenditure burden induced by high fertility is not sufficient to produce low savings rates as observed in the data. This may be because the model does not incorporate another channel through which fertility might affect saving, namely labor supply: high fertility may induce parents to work less or accumulate less human capital and thus lower saving. The model also tends to overpredict savings rates for high longevity countries. Absent from the model is the explicit form of the expenditure burden of the elderly. Much like fertility, longevity creates dependency burden to the economy and withdraw resources from productive purpose. Incorporating the labor supply effect of fertility as well as the expenditure burden of the elderly to the model and drawing its quantitative implications on cross-country savings rates may be worth pursuing.

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6 Appendix

6.1 Data Description

The savings data used are gross domestic savings rates from the World Development Indicators and private savings rates from the World Saving Database, both published by the World Bank. Attention is restricted to countries where time series data on savings rates are available for at least 10 years during the period 1980 2000. Excluded as well are countries with labor force less than 100 thousand during the period. The 19802000 period provides a relatively stable series on savings rates.

1. Gross Domestic Savings Rates

$$GDSR = \frac{GDS}{GDP} = \frac{GDP - C - G}{GDP},$$

where GDS is Gross Domestic Savings, GDP is Gross Domestic Products, C is aggregate private consumption and G is aggregate public consumption.

2. Private Savings Rates, PSR

$$PSR = \frac{GDS + (NFI - CGS)}{GDP + NFI},$$

NFI is net factor income from abroad plus international transfers and CGS is Central Government Savings. Notice that private savings rates are not always less than gross domestic savings rates. $PSR = GDSR$ if $NFI = CGS = 0$, $PSR < GDSR$ if $NFI < CGS$ and $PSR > GDSR$ if $CGS < 0$.

3. Survival Rates
Survival rates are obtained from WHO's lifetable. The table provides the death rate in 5-year intervals for the year of 2000. To construct the survival rates for the age groups of 20-40 and 40-60, I do the following. First, I obtain the mortality rate for each age group. Denote the mortality as M_x^n which means the mortality rate of the population living in the interval $(x, x+n)$. Next is to obtain the probability of death of the population living in the interval $(x, x+n)$, which is denoted as q_x^n and given by

$$q_x^n = \frac{nM_x^n}{1 + [n - a_x^n]M_x^n},$$

where a_x^n is the separation factor which, in this case, is 10. The survival rate, p_x^n, is $1 - q_x^n$. For p_1, we first calculate survival rate for age groups 0-1, 1-4 and 5-19 or p_0^1, p_1^4 and p_5^{19}. We use separation factors of 0.05 for age group 0-1 and 1.524 for age group 1-4. Then p_1 is given by $p_0^{19} = p_0^1 \times p_1^4 \times p_5^{19}$.

4. Fertility Rates
The data is taken from the World Development Indicators, 2004 (World Bank). Total

fertility rate represents the number of children that would be born to a woman if she were to live to the end of her childbearing years and bear children in accordance with prevailing age-specific fertility rates. For the computation of steady state savings rates, we divide fertility by 2 because young agents in our model are either males or females.

5. Income per Worker

The data is taken from the Penn World Table version 6.0.

6. Labor Force Growth Rate

The source is the World Development Indicators. We calculate the geometric growth rate between the year of 1980 and 2000.

6.2 Country List and Data

	Country	p1	p2	p3	n	g	Gross savings rate (%)
1	Albania	0.956	0.967	0.904	1.426	0.013	8.470
2	Algeria	0.939	0.967	0.897	2.195	0.039	31.435
3	Angola	0.663	0.723	0.599	3.550	0.024	24.902
4	Argentina	0.974	0.974	0.894	1.412	0.015	19.498
5	Australia	0.990	0.982	0.947	0.927	0.019	22.930
6	Austria	0.990	0.985	0.925	0.732	0.004	23.710
7	Bahrain	0.984	0.981	0.927	1.831	0.044	35.335
8	Bangladesh	0.896	0.938	0.824	2.073	0.024	10.868
9	Barbados	0.981	0.968	0.898	0.887	0.014	18.247
10	Belgium	0.990	0.981	0.925	0.797	0.003	22.135
11	Belize	0.957	0.969	0.888	2.141	0.035	15.129
12	Benin	0.817	0.870	0.755	3.168	0.027	0.975
13	Bolivia	0.899	0.941	0.827	2.321	0.026	11.752
14	Botswana	0.869	0.502	0.517	2.458	0.031	37.490
15	Brazil	0.945	0.955	0.868	1.362	0.026	21.693
16	Bulgaria	0.978	0.976	0.868	0.803	-0.005	23.968
17	Burkina Faso	0.755	0.748	0.646	3.477	0.018	2.608
18	Cameroon	0.830	0.812	0.713	2.850	0.027	21.432
19	Canada	0.991	0.985	0.941	0.832	0.016	22.598
20	Central African Republic	0.776	0.705	0.625	2.636	0.018	1.580
21	Chile	0.986	0.978	0.922	1.236	0.025	22.189
22	China	0.951	0.975	0.904	1.060	0.017	37.878
23	Colombia	0.963	0.939	0.894	1.510	0.034	19.109
24	Congo, Dem. Rep.	0.755	0.778	0.666	3.350	0.023	9.606
25	Congo, Rep.	0.865	0.802	0.721	3.145	0.031	31.693
26	Costa Rica	0.983	0.978	0.929	1.508	0.034	19.753
27	Cote d'Ivoire	0.803	0.758	0.660	2.995	0.034	18.665
28	Cyprus	0.987	0.981	0.940	1.125	0.013	19.434
29	Denmark	0.991	0.985	0.914	0.819	0.004	22.915
30	Dominican Republic	0.941	0.937	0.863	1.655	0.028	15.014
31	Ecuador	0.950	0.950	0.880	1.816	0.033	19.516
32	Egypt, Arab Rep.	0.941	0.970	0.858	1.986	0.027	14.007
33	El Salvador	0.952	0.941	0.859	1.867	0.026	5.013
34	Ethiopia	0.792	0.803	0.696	3.271	0.025	5.809
35	Fiji	0.965	0.963	0.866	1.535	0.023	14.389
36	Finland	0.992	0.980	0.919	0.868	0.004	27.104
37	France	0.991	0.981	0.924	0.902	0.006	20.917
38	Gabon	0.890	0.894	0.788	2.340	0.022	42.288
39	Gambia, The	0.864	0.920	0.802	2.883	0.036	5.576

	Country	p1	p2	p3	n	g	Gross savings rate (%)
40	Germany	0.991	0.984	0.924	0.689	0.004	21.809
41	Ghana	0.874	0.885	0.780	2.704	0.032	6.131
42	Greece	0.989	0.983	0.936	0.740	0.012	15.500
43	Guatemala	0.934	0.935	0.846	2.640	0.029	9.420
44	Guinea	0.808	0.869	0.746	2.879	0.024	15.931
45	Guyana	0.925	0.939	0.822	1.323	0.012	16.962
46	Hungary	0.985	0.975	0.832	0.835	-0.003	25.128
47	Iceland	0.992	0.987	0.948	1.067	0.015	20.888
48	India	0.882	0.934	0.819	1.880	0.020	21.091
49	Indonesia	0.935	0.947	0.834	1.543	0.026	30.625
50	Iran, Islamic Rep.	0.944	0.964	0.875	2.165	0.031	24.024
51	Ireland	0.989	0.983	0.930	1.108	0.012	24.789
52	Israel	0.990	0.982	0.942	1.436	0.031	9.137
53	Italy	0.991	0.985	0.940	0.659	0.006	22.658
54	Jamaica	0.978	0.974	0.895	1.449	0.015	18.490
55	Japan	0.993	0.988	0.939	0.768	0.009	31.073
56	Kenya	0.863	0.742	0.676	2.870	0.035	14.700
57	Korea, Rep.	0.986	0.979	0.906	0.898	0.022	32.842
58	Madagascar	0.827	0.912	0.783	3.027	0.026	3.765
59	Malawi	0.710	0.641	0.559	3.434	0.024	7.513
60	Malaysia	0.982	0.970	0.893	1.812	0.032	36.011
61	Mali	0.740	0.832	0.696	3.400	0.023	3.910
62	Malta	0.989	0.989	0.945	0.952	0.006	17.665
63	Mauritania	0.802	0.892	0.761	2.827	0.021	5.780
64	Mauritius	0.977	0.972	0.871	1.099	0.021	22.198
65	Mexico	0.966	0.970	0.901	1.599	0.031	23.349
66	Mongolia	0.916	0.945	0.843	1.917	0.023	16.912
67	Morocco	0.934	0.976	0.908	1.926	0.025	16.466
68	Namibia	0.883	0.724	0.676	2.678	0.029	11.839
69	Nepal	0.868	0.918	0.784	2.555	0.020	11.674
70	Netherlands	0.991	0.987	0.933	0.794	0.014	25.296
71	New Zealand	0.988	0.982	0.937	1.012	0.019	22.092
72	Niger	0.707	0.819	0.682	3.795	0.031	4.922
73	Nigeria	0.818	0.859	0.742	3.178	0.027	21.321
74	Norway	0.992	0.982	0.938	0.910	0.009	31.002
75	Oman	0.970	0.973	0.889	3.421	0.037	33.167
76	Pakistan	0.874	0.950	0.845	2.845	0.026	11.847
77	Panama	0.967	0.966	0.925	1.472	0.029	23.111
78	Papua New Guinea	0.880	0.927	0.804	2.617	0.026	19.603

	Country	p1	p2	p3	n	g	Gross savings rate (%)
79	Paraguay	0.958	0.969	0.891	2.263	0.028	14.763
80	Peru	0.938	0.955	0.876	1.755	0.030	21.526
81	Philippines	0.946	0.953	0.849	2.003	0.028	18.791
82	Poland	0.987	0.977	0.883	0.935	0.004	24.305
83	Portugal	0.987	0.974	0.919	0.796	0.006	19.362
84	Romania	0.970	0.975	0.862	0.868	-0.001	20.565
85	Saudi Arabia	0.961	0.970	0.881	3.170	0.041	27.496
86	Senegal	0.840	0.909	0.786	2.975	0.026	5.577
87	Sierra Leone	0.636	0.648	0.546	3.138	0.020	5.633
88	Singapore	0.993	0.987	0.946	0.830	0.030	45.137
89	South Africa	0.884	0.730	0.678	1.668	0.030	23.586
90	Spain	0.991	0.982	0.934	0.706	0.012	22.260
91	Sri Lanka	0.969	0.945	0.852	1.251	0.021	14.548
92	Sudan	0.857	0.899	0.783	2.635	0.028	7.265
93	Swaziland	0.837	0.588	0.561	2.578	0.033	6.791
94	Sweden	0.993	0.988	0.941	0.911	0.007	22.644
95	Switzerland	0.991	0.985	0.941	0.763	0.011	25.658
96	Syrian Arab Republic	0.965	0.969	0.887	2.603	0.037	14.557
97	Thailand	0.955	0.919	0.870	1.133	0.019	30.894
98	Togo	0.835	0.834	0.731	3.090	0.030	9.582
99	Trinidad and Tobago	0.970	0.958	0.861	1.162	0.016	26.507
100	Tunisia	0.960	0.972	0.897	1.658	0.029	23.295
101	Turkey	0.950	0.976	0.874	1.558	0.027	18.351
102	Uganda	0.814	0.741	0.631	3.407	0.027	3.437
103	United Kingdom	0.990	0.983	0.925	0.882	0.005	17.207
104	United States	0.987	0.976	0.916	0.984	0.014	17.382
105	Uruguay	0.979	0.976	0.896	1.224	0.013	16.069
106	Venezuela, RB	0.967	0.958	0.907	1.670	0.033	25.164
107	Yemen, Rep.	0.870	0.933	0.816	3.553	0.038	8.863
108	Zambia	0.770	0.558	0.511	3.048	0.026	10.443
109	Zimbabwe	0.841	0.474	0.437	2.365	0.029	16.416

Note: $p1, p2, p3$ are the the survival rates of individuals aged 0-15, 15-60 and more than 60 years, respectively; n is fertility rate, and g is the average of labor force growth rate over the period of $1980 - 2000$.

Table 1: Baseline Parameter Values

Parameter	Explanation	Value
α	share of physical capital	1/3
σ	coefficient of relative risk aversion	1.1
γ	fraction of income for child-rearing	0.10
δ	depreciation of physical capital	0.65
n	fertility per young worker	country specific
p_1, p_2, p_3	survival rates	country specific
g	labor force growth rate	country specific

Table 2: Data and Model Prediction on Gross Domestic Savings Rates under Baseline Parameter Values

Statistics	Data	Baseline Parameter
Average	18.7	18.7
Std. of Deviation	9.2	4.4
Minimum	1.0	7.4
Maximum	45.1	28.1
R^2		0.20
90^{th} to 10^{th} Percentile Ratio	5.4	1.9

Table 3: Data and Model Prediction on Private Savings Rates under Baseline Parameter Values

Statistics	Data	Baseline Parameter
Average	16.9	16.9
Std. of Deviation	7.4	4.2
Minimum	-0.2	8.0
Maximum	35.2	25.5
R^2		0.25
90^{th} to 10^{th} Percentile Ratio	3.8	1.9

Table 4: Data and Model Prediction on Gross Domestic Savings Rates under Baseline and Alternative Sets of Parameter Values

Statistics	Data	Baseline Parameter	Alternative 1 $\gamma : 0.1 \Rightarrow 0.15$	Alternative 2 $\gamma : 0.1 \Rightarrow 0.15$ $\delta : 0.05 \Rightarrow 0.1$
Average	18.7	18.7	18.7	18.7
Std. of Deviation	9.2	4.4	5.3	5.7
Minimum	1.0	7.4	6.1	5.9
Maximum	45.1	28.1	30.0	29.1
R^2		0.20	0.25	0.26
90^{th} to 10^{th} Percentile Ratio	5.4	1.9	2.5	2.6

Table 5: Data and Model Prediction on Private Savings Rates under Baseline and Alternative Sets of Parameter Values

Statistics	Data	Baseline Parameter	Alternative 1 $\gamma : 0.1 \Rightarrow 0.15$	Alternative 2 $\gamma : 0.1 \Rightarrow 0.15$ $\delta : 0.05 \Rightarrow 0.1$
Average	16.9	16.9	16.9	16.9
Std. of Deviation	7.4	4.2	5.1	5.5
Minimum	-0.2	8.0	5.7	4.9
Maximum	35.2	25.5	27.2	26.6
R^2		0.25	0.38	0.40
90^{th} to 10^{th} Percentile Ratio	3.8	1.9	2.6	2.7

Table 6: Fertility-only vs Survival-only Models Under Baseline Parameter Values

Statistics	Data	Baseline	Fertility only	Survival only
		$\gamma = 0.1$	$\beta = 0.74$	$\beta = 0.72$
Average	18.7	18.7	18.7	18.7
Std. of Deviation	9.2	4.4	2.2	4.5
Minimum	1.0	7.4	13.7	4.9
Maximum	45.1	28.1	25.0	23.0
R^2		0.20	0.13	0.18
90^{th} to 10^{th} Percentile Ratio	5.4	1.9	1.3	1.9

Table 7: Fertility-only vs Survival-only Models Under Alternative Parameter Values

Statistics	Data	Alternative 1	Fertility only	Survival only
		$\gamma = 0.15$	$\beta = 0.63$	$\beta = 0.59$
Average	18.7	18.7	18.7	18.7
Std. of Deviation	9.2	4.4	3.1	3.8
Minimum	1.0	7.4	12.2	4.8
Maximum	45.1	28.1	25.7	25.6
R^2		0.20	0.31	0.0.1
90^{th} to 10^{th} Percentile Ratio	5.4	1.9	1.6	1.6

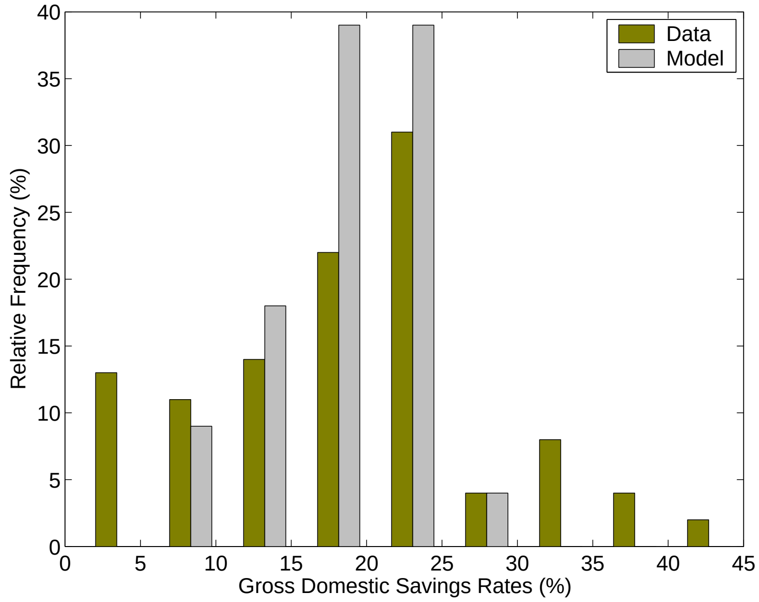

Figure 1: Distribution of Average Gross Domestic Savings Rates

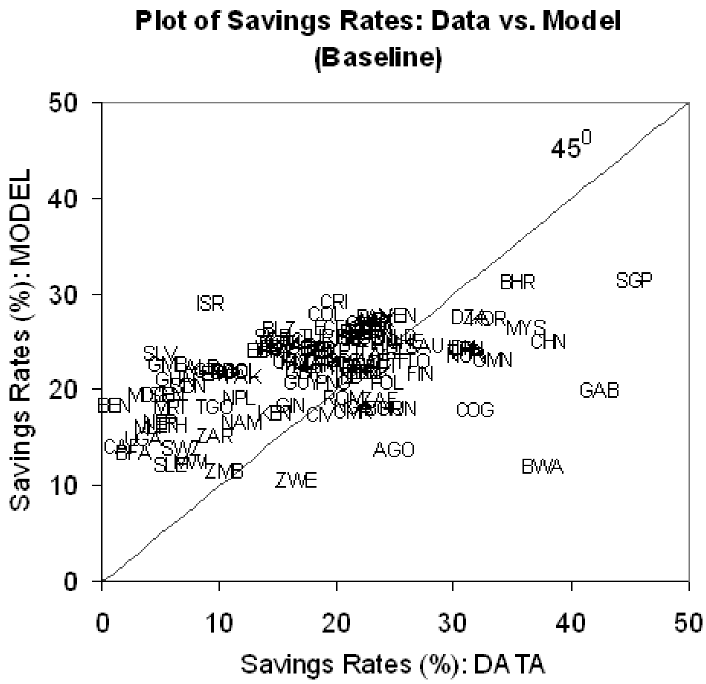

Figure 2: Scattered Plot of Savings Rates: Data vs. Model

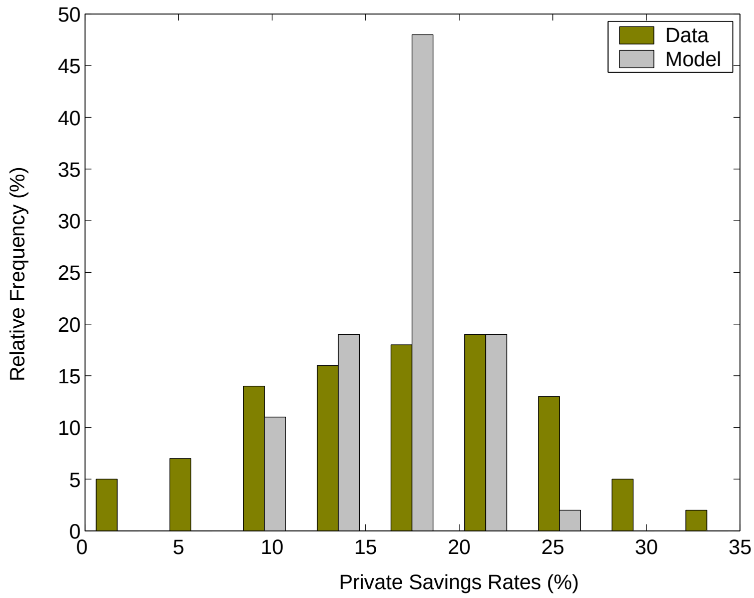

Figure 3: Distribution of Average Private Savings Rates

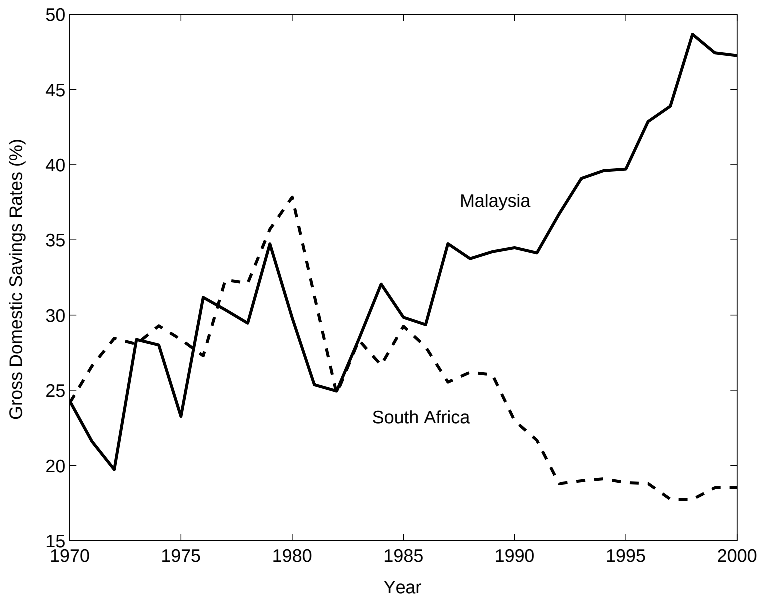

Figure 4: Gross Domestic Savings Rates in Malaysia and South Africa, 1970 – 2000

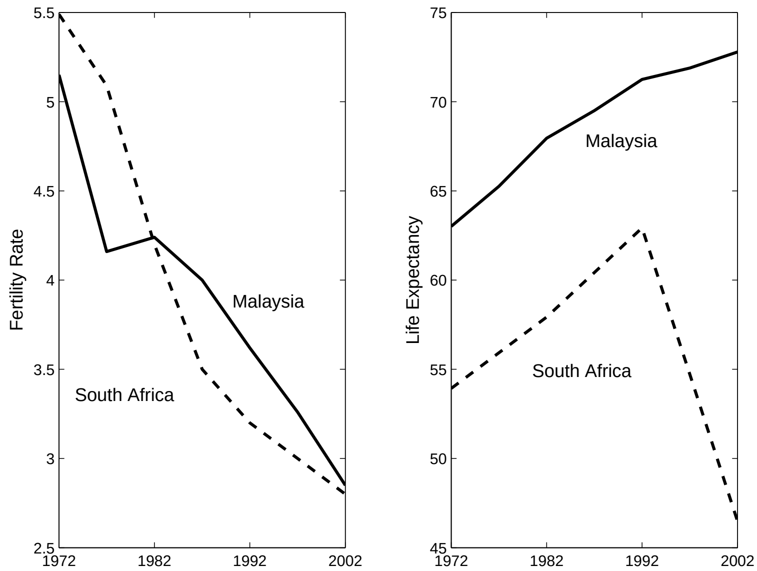

Figure 5: Fertility and Life Expectancy in Malaysia and South Africa, 1972 − 2002